Calculating Input Impedance of Electrically Small Insulated Antennas for Microwave Hyperthermia

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***Abstract*— Two analytical methods for approximating the input impedance of insulated monopole or dipole antennas embedded within an electrically dense medium have been reported in the literature. The methods differ by the applied degree of approximation in the solution of the integral equation for the current in the insulated conductor. These methods directly affect the calculation of the wavenumber and the characteristic impedance of an antenna treated as a lossy coaxial line. In the more complex approach the resulting formulas contain an additional term which improves the correlation with measured and numerically modeled results for electrically longer antennas. When applied to electrically small antennas (i.e. $< 1/8$ wavelength in the medium), this term introduces a significant error into the calculation of the real part of the complex input impedance. Special care must be taken if these formulas are used to design multisectional antennas in order to avoid impedance mismatch. Two methods for correcting this error are presented.**

I. Introduction

Insulated dipole and monopole antennas embedded in a dissipative dielectric medium may be found in many technical areas including communications, physical measurements, geophysical exploration and localized heating. An important medical application of localized heating is hyperthermic therapy of cancer and certain benign diseases [2]–[5], [9] where insulated antennas are often used as interstitial and intracavitary hyperthermia applicators.

Two analytical approximations which describe the current distribution and input impedance of these antennas have been developed by King *et al.* [6]–[8]. We will refer to these approximations as the *insulated antenna theory* (IAT). According to the IAT, the insulated antenna embedded in a lossy dielectrtic is treated as a lossy transmission line terminated with an ideal open end which assumes infinite terminating impedance. The transmission parameters of the line are derived from approximate solutions of the integral equation for the current along the antenna.

The methods differ bya the applied degree of approximation in the solution of the integral equation for the current in the insulated conductor and also differ slightly in assumptions. These results directly affect calculation of the wavenumber of the current and the characteristic impedance of an antenna treated as a lossy coaxial line. The simpler approximation [7], although appropriate for electrically smaller antennas ($h < 1/4$ wavelength in the surrounding medium—λ_M) leads to significant error as h becomes longer. The more complex (MC) approach [7] contains an additional term which improves the correlation with measured and numerically modeled results for electrically longer antennas. When applied to electrically small antennas (i.e. $< 1/8\lambda_M$), this additional term introduces a significant error into the calculation of the real part of the complex input impedance.

Iskander and Tumeh [3] have proposed an iterative approach to designing multisectional antennas based on the MC method of approximating the current distribution. If their approach is used to develop devices (such as hyperthermia applicators) containing multisectional antennas special care must be taken to avoid impedance matching error.

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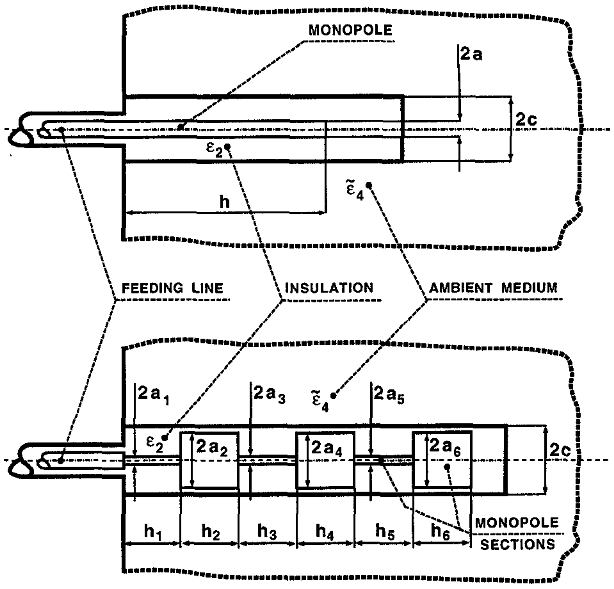

Fig. 1. Insulated antenna embedded in an electrically dense medium. Uniform monopole (at top). Sectorial antenna in the form used in Table I (at bottom).

II. Methods

The IAT was used to compare the two approximation methods for electrically small antennas. Consider the case of a monopole antenna with lossless insulation and time dependence $e^{j\omega t}$. The insulated monopole (Fig. 1(a) consists of a perfect central conductor of length h and outer diameter (OD) $2a$. This is surrounded by an insulating layer with OD = $2c$. The monopole is assumed to be ideally open ended. The insulation may consist of one or two layers. However, for simplicity, only a single layer will be considered. Outside the insulation there is an infinite ambient medium. The wavenumber of the insulating layer is $k_2 = \omega(\mu_0\epsilon_2)^{1/2}$ and the wavenumber of the ambient medium is $k_4 = \beta_4 - j\alpha_4 = \omega(\mu_0\tilde{\epsilon}_4)^{1/2}$.

The assumptions of the IAT are that the cross section of the antenna is electrically small ($(k_2c)^2 \ll 1$) and that the electric density ($|k_i|$) of the ambient medium is higher than that of the insulation. In the MC case $|k_4/k_2|^2 \geq 2$ and in the less complex (LC) approximation $|k_4/k_2|^2 \geq 16$.

The input impedance of the monopole antenna can be calculated from the complex wavenumber (k_L) and the complex characteristic impedance Z_c of the line:

$$Z_{in} = -jZ_c/\tan(k_L h) \tag{1}$$

In the case of LC approximation of the current distribution:

$$k_L = k_2[1 + F/\ln(c/a)]^{1/2} \tag{2}$$

$$Z_c = k_L \ln(c/a)/(2\pi\omega\epsilon_2) \tag{3}$$

where $F = H_0^{(2)}(k_4c)/[k_4cH_1^{(2)}(k_4c)]$. The MC approach yields:

$$k_L = k_2[1 + F/\ln(c/a)]^{1/2}[1 + (F(k_2/k_4)^2/\ln(c/a))]^{-1/2} \tag{4}$$

$$Z_c = k_L \ln(c/a)/(2\pi\omega\epsilon_2) + k_L F(k_2/k_4)^2/(2\pi\omega\epsilon_2) \tag{5}$$

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TABLE I
CALCULATED INPUT IMPEDANCE OF SECTIONS OF A SHORT (12 mm) APPLICATOR IN TWO DIFFERENT MEDIA, CALCULATED WITH THE MC AND THE LC RELATIONS (SEE TEXT FOR EXPLANATION)

f = 433 MHz, 2c = 1 mm ε_2 = 2.34							
Section No (i)		1	2	3	4	5	6
$2a_i$ [mm]		0.2	0.6	0.2	0.6	0.2	0.6
h_i [mm]		2	2	2	2	2	2
$\tilde{\varepsilon}_4$ = 78.0 - j1.6 (H_2O at 25°C)							
Re $Z^{(i)}_{in}$	MC*	-9.01	-12.07	-16.58	-22.44	-36.92	-62.29
	LC	3.91	2.75	2.74	1.61	1.64	0.56
Im $Z^{(i)}_{in}$	MC	-201.2	-225.3	-310.6	-362.5	-632.2	-863.0
	LC	-170.5	-191.3	-266.0	-307.4	-543.2	-720.8
$\tilde{\varepsilon}_4$ = 52.8 - j52.0 (MUSCLE)							
Re $Z^{(i)}_{in}$	MC*	16.82	18.09	22.87	27.21	42.82	68.36
	LC	2.94	2.08	2.07	1.21	1.24	0.42
Im $Z^{(i)}_{in}$	MC	-201.4	-225.6	-310.9	-362.9	-632.9	-864.2
	LC	-171.4	-191.2	-266.0	-307.3	-543.2	-720.8

*) marked ⊙ on Fig 2

In the case of multisectional IAT [3], the antenna (Fig. 1(b)) consists of a central conductor divided into n sections of OD = $2a_i$ (where $i = 1 \cdots n$), length h_i and uniform insulation of OD = $2c$. The input impedance for i-th section is:

$$Z_{\text{in(i)}} = jZ_{c(i)} \coth(k_{L(i)} h_i + j\theta_i) \quad (6)$$

where $\theta_i = \coth^{-1}(Z_{\text{in(i+1)}}/Z_{c(i)})$ is the angle used to take into account the termination of the ith section with the load impedance $Z_{c(i+1)}$. The input impedance of the entire applicator is given by $Z_{\text{in}(1)}$.

III. RESULTS AND DISCUSSION

Consider the task of designing a thin, single section hyperthermia applicator which is to operate at 434 MHz with OD = $2c = 1$ mm and $2a = 0.2$ mm. When the ambient medium is muscle equivalent ($\tilde{\varepsilon}_4 = 52.8 - j52$) the real part of the input impedance, for the MC case, approaches infinity as h approaches zero (the solid line in Fig. 2). When the ambient medium is distilled water ($\tilde{\varepsilon}_4 = 78 - j1.6$ at 25°C), the input impedance becomes infinitely negative (Fig. 3.) These results are in error for small h since as h approaches 0, the real part of the input impedance should also approach zero while the imaginary part should become infinite. Interestingly, it is possible to define a hypothetical ambient medium ($\tilde{\varepsilon}_4 = 60 - j24.03$) which demonstrates correct behavior (dashed line in Figs. 2 and 3) for h over the range illustrated. Correct behavior may also be obtained for small h by using the LC approximation of current distribution (dashed line in Figs. 2 and 3).

The case of a short ($h \approx \lambda_M/8$) 6-section antenna (Fig. 1(b)) is presented in Table I. Values for the real (Re) and imaginary (Im) parts of the input impedance (Z_{in}) of each section are given for ambient media of muscle and water using both methods of approximating the current distribution. For the MC case, the values of Re $Z_{\text{in}(1)}$ have also been plotted in Figs. 2 and 3. Note that as h becomes longer, the error decreases. Table II presents $Z_{\text{in}(1)}$ values for a longer ($h \approx \lambda_M/2$), 5-section antenna in ambient media of water, muscle, and a theoretical lossless medium.

Negative values for the real part of the input impedance appear in two cases and there is no correlation between Re or Im Z_{in} values for the two methods of approximating the current distribution. Neither method of approximating the current distribution gives satisfactionary results over a wide range of h.

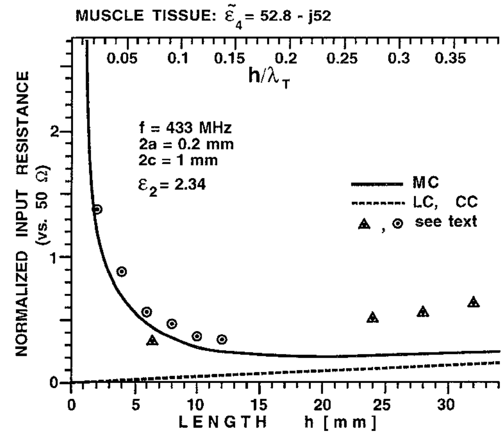

Fig. 2. Calculated input resistance (real part of the input impedance, Re Z_{in}) vs. the length of an insulated monopole in a muscle equivalent medium for the more complex (MC), less complex (LC) and corrected (CC) approaches.

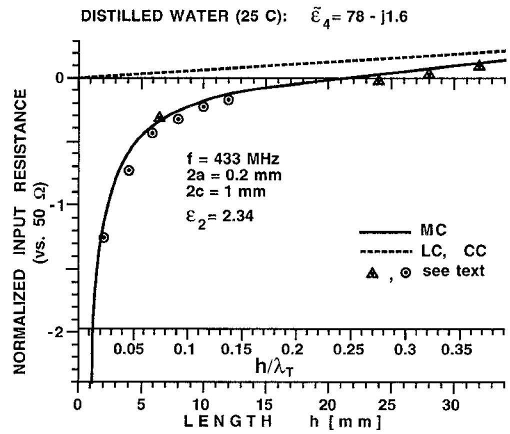

Fig. 3. Calculated inpout resistance (Re Z_{in}) vs. the length of an insulated monopole in water for the more complex (MC), less complex (LC) and corrected (CC) approaches.

The formulas for k_L and Z_c introduce a certain amount of error into the approximation of Re Z_{in}. In the MC case this effect can be neglected for longer $h(h > \lambda_M/4)$. For smaller $h(h < \lambda_M/8)$ the calculated values of Re Z_{in} can be a problem. Formula (1) is a classic relation for an ideally open ended transmission line. Both k_L and Z_c are related to the transmission line parameters R, L, G and C. For lossless insulation $G = 0$ and the argument of the complex values of k_L and Z_c are equal.

In the LC approximation with $G = 0$ the argument of k_L and Z_c will be equal regardless of the accuracy of formulas (2) and (3) owing to the linearity of the formulas. In the MC case, the argument terms (ϕ_Z and ϕ_k) of $Z_c = |Z_c| \exp\{j\phi_Z\}$ and $k_L = |k_L| \exp\{j\phi_k\}$ will generally differ because the term $F(k_2/k_4)^2$ in relation (5) is generally complex. The expression for Re Z_{in}, calculated from (1), is quite sensitive to variation of $\phi_Z - \phi_k$ when h is small. If the insulation is lossless:

$$\lim_{h \to 0} \{\text{Re}/, Z_{\text{in}}\} = \begin{cases} +\infty & \text{for } \phi_Z > \phi_k \\ 0 & \text{for } \phi_Z = \phi_k \\ -\infty & \text{for } \phi_Z < \phi_k \end{cases}$$

One possible method of obtaining more accurate results for small h would be to use MC equation (4) to calculate k_L and to substitute this result into LC equation (3) to calculate Z_c. This method would provide a more accurate approximation of k_L for longer h while

TABLE II
COMPARISON OF THE INPUT IMPEDANCES, CALCULATED WITH THE MC AND THE LC RELATIONS, FOR A 5-SECTIONAL 40 MM LONG APPLICATOR EMBEDDED IN THREE DIFFERENT MEDIA (SEE TEXT FOR EXPLANATION)

f = 433 MHz, 2c = 1 mm ε_2 = 2.54						
Section No (i)		1	2	3	4	5
$2a_i$ [mm]		1.6	1.4	1.3	0.1	2.0
h_i [mm]		8.0	4.0	4.0	17.5	6.5
$\tilde{\varepsilon}_4$ = 55.0 - j0.0						
$Re\ Z_{in}^{(i)}$	MC	4.79	-2.09	-5.43	-9.01	-26.76
	LC	160.0	49.07	27.29	17.46	1.95
$Im\ Z_{in}^{(i)}$	MC	19.13	4.77	-1.98	-8.98	-67.04
	LC	-15.90	67.37	51.67	39.57	-16.18
$\tilde{\varepsilon}_4$ = 78.0 - j1.6 (H_2O at 25°C)						
$Re\ Z_{in}^{(i)}$	MC*	15.98	5.22	1.63	-1.68	-17.39
	LC	147.3	44.40	26.46	17.18	1.92
$Im\ Z_{in}^{(i)}$	MC	37.03	18.49	11.33	4.52	-50.32
	LC	19.59	62.12	48.62	37.56	-16.40
$\tilde{\varepsilon}_4$ = 52.8 - j52.0 (MUSCLE)						
$Re\ Z_{in}^{(i)}$	MC*	43.00	31.32	27.37	24.36	15.38
	LC	189.8	34.29	20.00	12.90	1.44
$Im\ Z_{in}^{(i)}$	MC	12.56	10.97	6.90	1.93	-53.00
	LC	0.59	67.25	50.36	38.21	-16.33

*) marked ⚠ on Fig 2

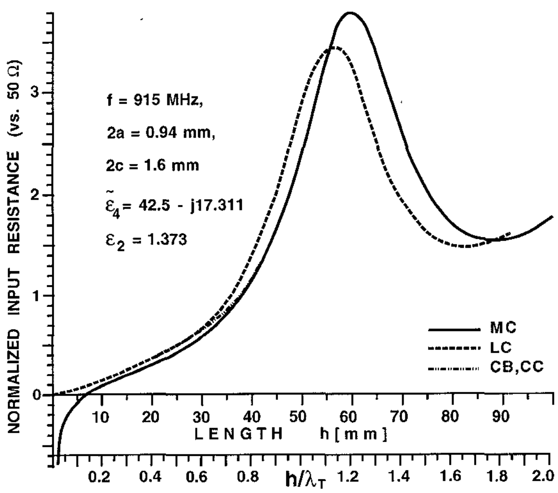

Fig. 4. Comparison of input resistance (Re Z_{in}) for a theoretical 915 MHz hyperthermia applicator similar to the one reported in [8], computed for the more complex (MC), less complex (LC) and corrected (CC \& CB) approaches. Note that the CB and CC curve begins to diverge from LC as h increases, subsequently coinciding with the MC curve (see text for explanation).

reducing overall error in the calculation of Z_{in} for small h. For longer h, however, significant error would remain in the calculation of Z_{in} owing to the use of the LC formula. Another possibility is to modify the MC formulas by substituting the modulus of term $F(k_2/k_4)^2$ in formula (5). This would be expected to improve the accuracy of Z_{in} for small h at the expense of accuracy for longer h. Calculation of Z_{in} using this approach is plotted in Fig. 4 (CB). Good accuracy is demonstrated for Re Z_{in} over a wide range of h. However, for longer h some error remains in Im Z_{in} (see Fig. 5, CB).

A third possibility is to use the MC equations to calculate Z_c and k_L and to introduce a new term into (1) which would affect only the real part of Z_{in} for small h. If h and the argument $(\phi_Z - \phi_k)$ are

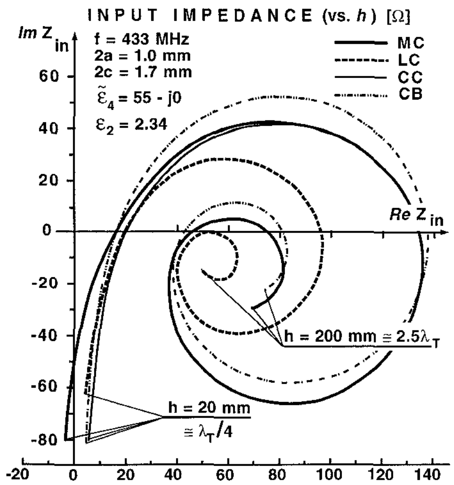

Fig. 5. Calculated input impedance of a theoretical 433 MHz hyperthermia applicator on the complex plane, computed as in Fig. 4. Note that the MC curve is known to provide the most accurate results for long h [7], but fails to correctly approach zero for small h. The corrected CC curve coincides with the MC curve for long h while maintaining proper behavior for small h.

small then the real part of the input impedance obtained from (1) is

$$\mathrm{Re}Z_{in} = |Z_c|(\phi_Z - \phi_k)/(|k_L|h). \quad (7)$$

For the general assumption of lossless insulation ($G = 0 \to \phi_Z = \phi_k$) the above expression reduces to zero. In the case of $\phi_Z \neq \phi_k$ this term can be considered to approximate the error for small h while tending to zero as $h \to \infty$. Introducing this term (7) into (1) results in

$$Z_{in} = -jZ_c/\tan(k_L h) - |Z_c|(\phi_Z - \phi_k)/(|k_L|h). \quad (8)$$

Calculation of Z_{in} using this approach is plotted in Figs. 2, 3, 4, and 5 (CC). In Figs. 2 and 3 the CC plots (for Re Z_{in}) agree with the LC case. In Fig. 4 the Re Z_{in} CC curve coincides with LC for small h and smoothly transitions into the MC case for longer h. In Fig. 5 the CC curve is observed to behave correctly for both real and imaginary parts of Z_{in} for all h.

In some situations, such as the design of hyperthermia applicators containing sectorial antennas, it may be desirable to use the second error correction method (CB) even though it is less accurate than the later (CC) method. Using the CC method would require that a new equation for $Z_{in(i)}$ be developed whereas the CB method would use the existing equation (6).

IV. CONCLUSIONS

The existing IAT has been demonstrated to be inappropriate for electrically short antennas. Using the existing formulae leads to incorrect results when designing multi-sectional antennas with "short" sections. The most significant source of the error has been traced to the mathematical properties of the function used to calculate the input impedance. We have proposed three methods of correcting this error, the third of which appears to be quite promising.

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